Games and Economic Behavior **28,** 55–72 (1999)
Article ID game.1998.0689, available online at http://www.idealibrary.com on IDEAL®

Distributed Games*

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Received September 9, 1997

The Internet exhibits forms of interactions which are not captured by existing models in economics and artificial intelligence. New models are needed to deal with these interactions. In this paper we present a new model—distributed games. In such a model each player controls a number of agents which participate in asynchronous parallel multiagent interactions (games). The agents jointly and strategically (partially) control the level of information monitoring and the level of recall by broadcasting messages. As an application, we show that the cooperative outcome of the Prisoner's Dilemma game can be obtained in equilibrium in such a setting. *Journal of Economic Literature* Classification Numbers: C72, C73, D83.

1. INTRODUCTION

The Internet introduces new challenges in artificial intelligence, economics, and game theory.[1] In particular, it exhibits both parallel and sequential interactions. While sequential interactions have been extensively discussed in the literature, the study of parallel interactions has been neglected so far. New models of economies and games are needed in order to effectively deal with parallel interactions. In this paper we present one such new model—distributed games. Our model captures several features of distributed systems, such as the Internet. Such systems are usually assumed to be asynchronous systems where agents communicate by broadcasting messages (Tanenbaum, 1988). To motivate our particular definition, think of

* First Version: April 1997. This work has been influenced by discussions made by both authors with the participants of the artificial intelligence (AAAI) Spring Symposium, Stanford University 1997, and with the participants of the international conference of game theory, University of New York at Stony Brook, 1997. We thank Abraham Neyman, Haim Reisman, Ariel Rubinstein, Dov Samet, Yoav Shoham, and Doron Sonsino for helpful discussions and suggestions.

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[1] See e.g., Varian (1995) and Boutilier *et al.* (1997).

0899-8256/99 $30.00

several users (players) who send software agents[2] to participate in auctions which are held in various locations of the net, and are conducted in a randomly chosen order. This random order of play models a situation where all auctions are supposed to take place in parallel but, due to the fact that real distributed systems are asynchronous, are actually held at different time intervals.[3]

The agents can communicate by broadcasting messages. A message sent by an agent of a particular user is not considered to be private information of the agents of this user.[4] In a general distributed game the agents may be engaged in any type of interaction at each of the locations. More precisely, a distributed game is defined by the following elements:

(1) A set of players.

(2) A set of locations.

(3) A set of agents for each player, one agent for each location.

(4) A set of games in strategic form with the given set of players, one for each location.

(5) A set of messages for each player.

(6) A probability distribution over the set of permutations of locations.

At the initial stage, Nature chooses a permutation according to the given probability distribution. In the following stages the games in the locations are played (by the agents) according to the order prescribed by the chosen permutation. After a game at a location is played, the agents at this location can send messages. The agents do not appear in the formal definition of the distributed game [which can be described formally as an extensive game with simultaneous moves (see, e.g., Osborne and Rubinstein, 1994) with the given set of players], however its information structure is motivated by their existence: When an agent is called to choose an action and right after that to send a message, it is aware of its location. Any additional information it can use must be extracted from the sequence of messages it receives. Thus, two histories which generate the same sequence of messages are in

[2]Roughly speaking, software agents are programs which are designed to serve the goals of a particular user. These agents may navigate in a computerized network, while transmitting messages among themselves, and interacting with other agents (which might be controlled by other users). The design of software agents is one of the most important research and application directions of the computer industry (CACM, 1994).

[3]It may also reflect a random decision of the auctions' organizers.

[4]One can relax this assumption by introducing probability of detection, which may be player specific. Our discussion and results hold also for networks where the content of messages can be encrypted, but the fact a message has been sent cannot be considered as private information.

the same information set. We assume that each message has a location and sender stamps, and that an agent can record the order of arrival of the messages. For example, consider a particular location. An agent at this location cannot distinguish between a history in which this location is the first location to a history in which this location is the last one and no agent in all other locations sent a message.

Hence, a distributed game is an extensive game with simultaneous moves, with incomplete information (e.g., about the order of stages) and with a specific player-symmetric information structure as described above. Such games have imperfect recall[5] because the particular information structure (motivated by the use of agents) implies that a player does not necessarily remember what were the previous locations at which he has already played, and even if he knows that he has already played at a particular location, he does not necessarily remember what were the actions that were selected at this location. Notice that in distributed games the players can strategically decide on the level of recall. This point will be discussed later.

A particular type of a distributed game is a parallel game which is defined by the additional condition that each permutation is equally likely.[6] This definition reflects the asynchronous nature of distributed systems. In a parallel game the interactions are supposed to be completely parallel, but due to the asynchronous nature of distributed systems there is a stochastic noise caused mainly by asynchronous clocks, implying sequential interactions. The players are ignorant about the precise nature of the noise and therefore all they can do is use the principle of indifference (Keynes, 1963). That is, they assign equal probabilities to all permutations of locations.[7] A distributed game is location-symmetric if the same stage game in strategic form is played at all locations.

As an application, we analyze a location-symmetric parallel game in which the same Prisoner's Dilemma game is played at all locations. We show that if the number of locations is sufficiently large, the outcome of cooperation can be obtained in equilibrium. In this equilibrium, an agent cooperates if and only if it has not gotten a message. If it detects a deviation from cooperation it immediately broadcasts an alarm message, otherwise it does not send any message. Note that this equilibrium survives due to the joint decision of agents not to reveal to their partners and opponents any

[5]See the discussion at the end of Section 2.

[6]That is, its probability is $1/n!$, where n denotes the number of locations.

[7]Parallel games model multiagent interactions in computerized distributed systems with asynchronous clocks (which cause the stochastic noise on order of stages); Each location is identified with a processor, and each agent which is active at this location, is identified with a process that runs in the processor. The assumption of asynchronous clocks is typical in distributed computing (see, e.g., Fagin *et al.* (1995) for a discussion of this issue in the context of the theory of knowledge).

information regarding the number of future interactions, as long as their opponents cooperate with them. Hence, in equilibrium the agents never know how many more games are to be played. This lack of knowledge enables cooperation.[8] This result is extended to a general folk theorem for location-symmetric parallel games.

2. DISTRIBUTED GAMES

A *distributed game* DG is defined by the following elements:

$DG1$ A finite set of players $F = \{1, 2, \ldots, m\}$, where $m \geq 2$.

$DG2$ A finite set of locations $L = \{1, \ldots, n\}$, $n \geq 1$.

$DG3$ For each location i a game G_i in strategic form, with the set of players F. The action set of player j at the game G_i is denoted by S_i^j. We assume that S_i^j contains at least two actions. The payoff function of player j at G_i is $u_i^j : S_i \to R$, where $S_i = \times_{j=1}^m S_i^j$.

$DG4$ To each player j, a set of agents $\{i_j\}_{i \in L}$; i_j is referred to as the agent of j at location i.

$DG5$ For every player j a finite set M^j of messages. We assume that M^j contains the no-message option, denoted by φ, and contains at least one real message. Thus for every player j, $M^j = \{\varphi, a_1^j, \ldots, a_{k^j}^j\}$, where $k^j \geq 1$. The agents of player j can broadcast messages from M^j.[9]

$DG6$ A probability distribution λ on the set PE of $n!$ permutations of $\{1, 2, \ldots, n\}$, where a permutation is a one to one function $r : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$.

We need the following definition. Consider a distributed game described by $DG1$–$DG6$. We define the message game MG to be the game in strategic form whose players are F. The action set of player j is M^j and the payoff functions of the players are defined on $M = \times_{j \in F} M^j$, and they are constantly zero. In the sequel we will refer to the notion of an empty message. The empty message is not a message, it is just a way to describe the event that an agent does not send a message. We now define the rules of move, the payoffs, and the information structure of the distributed game DG defined by $DG1$–$DG6$.

[8] Equilibrium of this type is discussed in Nishihara (1997) and in Neyman (1998) (see the discussion in Section 4).

[9] Our discussion and results can be easily extended to the case where each agent has its own set of messages.

The Rules of Move

The game has $n+1$ stages. At the initial stage (Stage 0). Nature chooses a permutation $r \in PE$ according to the probability distribution λ. Each additional stage $1 \leq t \leq n$ is divided into two sub-stages, the action stage $t[a]$ and the message stage $t[m]$. At stage $t[a]$ the players play the game $G_{r(t)}$, that is, they choose simultaneously an action profile $x_t \in S_{r(t)}$. At Stage $t[m]$, which comes after stage $t[a]$, the players play the game MG. That is, they choose simultaneously a message profile $m_t \in M$.

The Payoff Structure

If Nature chooses r, and the players choose the action profiles $x_t \in S_{r(t)}$, $1 \leq t \leq n$, then player j receives the accumulated payoffs in the stage games, that is, he receives $\sum_{t=1}^{n} u^j_{r(t)}(x_t)$.

The Information Structure

For each $r \in PE$ and for each $2 \leq T \leq n$, let $H^a_r[T] = \times_{t=1}^{T-1}(S_{r(t)} \times M)$ be the set of histories that can be generated in the game after the permutation r is chosen at stage 0 and just before the action selection stage $T[a]$. When $T = 1$ the agents at location $r(1)$ face the empty history denoted by e_r. Hence, $H^a_r[1] = \{e_r\}$. Let $H^a_r = \bigcup_{T=1}^{n} H^a_r[T]$. Similarly, let $H^m_r[T] = H^a_r[T] \times S_{r(T)}$ be the set of histories that can be generated in DG after the permutation r is chosen and just before the message selection stage $T[m]$. Let $H^m_r = \bigcup_{T=1}^{n} H^m_r[T]$, and let $H_r = H^a_r \cup H^m_r$.

Let $H = \{(r, h_r) : r \in PE, h_r \in H_r\}$. Every $h \in H$ is a non-null history in DG. An *information structure* of a player is a partition of H. Elements in this partition are called *information sets*. As all agents at a particular location receive the same information, the structure of information in a distributed game is player-symmetric. That is, we have to define only one partition Π which is common to all players. We define this partition by using the concept of signals. With each $h \in H$ we associate a signal $I(h)$. Two histories are in the same information set if and only if they are associated with the same signal. We refer to I as the *signaling function*. We first define the set Z_i of possible signals at location i. Consider an agent at location i just before an action stage. This agent knows its location. If it does not receive any message we denote the associated signal by (i, NM) ("no message at location i"). Otherwise the agent receives a signal of the form

$$z = \big(i, ((i_1, m_1), \ldots, (i_T, m_T))\big) \tag{2.1}$$

of non-null message profiles with a location and sender stamps. Thus, an agent that receives the signal z given in (2.1), knows that it is located at i, and that in the chosen permutation r, i_t comes before i_{t+1}, $1 \leq t < T$ and

i_T comes before i (that is, $r^{-1}(i_t) < r^{-1}(i_{t+1}) < r^{-1}(i)$). This agent further knows that for $1 \leq t \leq T$, the agent of player j at location i_t broadcasted the message m_t^j. Let Z_i^a be the set of all signals that are feasible for an agent at location i just before an action stage. That is, Z_i^a contains the signal (i, NM) and every signal that is described by (2.1). Similarly, an agent at location i, at a message selection stage receives a signal in Z_i^a attached with an action profile from S_i. Let $Z_i^m = Z_i^a \times S_i$ be the set of all signals that are feasible for an agent at location i just before a message selection stage. Let $Z_i = Z_i^a \cup Z_i^m$ and let $Z = \bigcup_{i=1}^n Z_i$. Z is the set of signals. We now define $I: H \to Z$ as follows: Let $h = (r, h_r) \in H$. Therefore, for some $1 \leq T \leq n$, $h_r \in H_r^a[T]$ or $h_r \in H_r^m[T]$. Denote $i = r(T)$. Assume $h_r \in H_r^a[T]$. If $h_r = e_r$, $I(h) = (i, NM)$. Otherwise, $h_r = (x_1, m_1), \ldots, (x_{T-1}, m_{T-1})$. Let $i_1 = r(t_1), \ldots, i_k = r(t_k)$, $1 \leq k \leq T-1$, be the r-ordered sequence of locations from which a non-null message profile is broadcasted. That is, let t_1 be the first t for which m_t is non-null, then $i_1 = r(t_1)$, and for $1 < q \leq k$, t_q is the first $t > t_{q-1}$ for which m_t is non-null, and $i_q = r(t_q)$. $I(h)$ is defined to be $(i, ((i_1, m_{t_1}), \ldots, (i_k, m_{t_k}))) \in Z_i^a$. If $h_r \in H_r^m[T]$, $h_r = (\bar{h}_r, x_T)$ with $\bar{h}_r \in H_r^a[T]$ and $x_T \in S_i$. In this case we define $I(h) = I(r, h_r) = (I(r, \bar{h}_r), x_T) \in Z_i^m$.

In the sequel we make use of the following additional notations and definitions: Every signal $z \in Z_i^a$ is a pair (i, y), where $i \in L$ and y is a sequence of messages with location and sender stamps. We denote the set of all such sequences by Y^a. That is, $Z_i^a = L \times Y^a$. An element of Y^a is called *a location-free signal at an action stage*.

An Intuitive Demonstration of the Information Structure

It is useful to demonstrate our definition of the information structure in distributed games by a simple example. Suppose there are four locations ($n = 4$), $L = \{1, 2, 3, 4\}$, and two players ($m = 2$). Denote by i_j the agent of player j at location i. Let $M^1 = M^2 = \{\varphi, A, B, \ldots Z\}$ be the message sets. Assume $i = 4$ and that the agents at location 4 are about to choose their actions in S_4 (Agent 4_j, $j \in \{1, 2\}$ has to choose an action in S_4^j). Agent 4_1 (as well as Agent 4_2) receives the signal

$$z = \big(i, \big((i_1, m_1), (i_2, m_2)\big)\big) = \big(4, \big((3, (A, B)), (1, (B, \varphi))\big)\big).$$

That is, $i_1 = 3$, $i_2 = 1$, $m_1 = (A, B)$, and $m_2 = (B, \varphi)$. This agent (as well as the other agent at location 4) deduces that in the chosen permutation r, location 3 comes before location 1, and location 1 comes before location 4. That is, it deduces that the chosen permutation is one of the 4 permutations (out of 24 possible permutations): $r^1 = 2314$, $r^2 = 3214$, $r^3 = 3124$ or $r^4 = 3142$. In addition, it deduces that the stage games at locations 3 and 1 have already been played. Note that if $r = r^1$, r^2, or r^3, then it means that the game at location 2 is over, but neither of the two agents 2_1 or 2_2 sent a message. If the true r is r^4, the game at location 2 will be played after the current stage is over. In addition, Agent 4_1 knows the messages from locations 3 and 1, that is, it knows that Agent 3_1 sent the message A, agent 3_2 sent the message B, Agent 1_1 sent the message B, and Agent 1_2 did not send a message. Note that the agents at location 4 may deduce nothing about the actions chosen at previously played locations. To conclude, these agents extract the following information about the true history (r, h_r):

(1) $r = 2314$ and $h_r = ((x_1, (\varphi, \varphi)), (x_2, (A, B)), (x_3, (B, \varphi)))$, for some x_1, x_2, x_3 in S_2, S_3, S_1 respectively, or

(2) $r = 3214$ and $h_r = ((x_1, (A, B)), (x_2, (\varphi, \varphi)), (x_3, (B, \varphi)))$, for some x_1, x_2, x_3 in S_3, S_2, S_1 respectively, or

(3) $r = 3124$ and $h_r = ((x_1, (A, B)), (x_2, (B, \varphi)), (x_3, (\varphi, \varphi)))$, for some x_1, x_2, x_3 in S_3, S_1, S_2, respectively, or

(4) $r = 3142$ and $h_r = ((x_1, (A, B)), (x_2, (B, \varphi)))$, for some x_1, x_2 in S_3, S_1, respectively.

We now define strategies in distributed games.

Strategies

A *strategy* for player j in DG is a pair $\sigma^j = (f^j, g^j)$, where $f^j = (f_i^j)_{i=1}^n$ and $g^j = (g_i^j)_{i=1}^n$, where $f_i^j\colon Z_i^a \to S_i^j$ and $g_i^j\colon Z_i^m \to M^j$. We refer to (f_i^j, g_i^j) as the strategy of Agent i_j. That is, for every information signal $z \in Z_i^a$ Agent i_j chooses the action $f_i^j(z)$, and for every information signal $z \in Z_i^a$ and $x_i \in S_i$ Agent i_j broadcasts the message $g_i^j(z, x_i)$.

Equilibrium

Let H_n be the the set of all histories of the form (r, h_r), where h_r is a sequence of pairs of action and message profiles of length n. That is, H_n is the set of all histories $\psi \in H$ of the form

$$\psi = \big(r, ((x_1, m_1), \ldots, (x_n, m_n))\big),$$

where $x_t \in S_{r(t)}$ and $m_t \in M$ for all $1 \le t \le n$. Every tuple of strategies $\sigma = (\sigma^j)_{j=1}^m$ defines a probability distribution μ_σ over H_n. The expectation operator with respect to μ_σ is denoted by E_σ. Let $\tilde{r}$ be the random variable on H_n defined by $\tilde{r}(\psi) = r$, and let X_i, $i \in L$ be the random variables defined by $X_i(\psi) = x_{r^{-1}(i)}$.

Let $E_i^j(\sigma) = E_\sigma(u_i^j(X_i))$, be the expected payoff of player j from location i when all players use the strategy profile σ. The expected payoff of player j when all players use the strategy profile σ is denoted by $u^j(\sigma)$. That is,

$$u^j(\sigma) = \sum_{i=1}^{n} E_i^j(\sigma).$$

Consider a distributed game DG given by $DG1$–$DG6$. We can associate with it a game in strategic form in which the player set is F, the action set of player j is the set Σ^j of the strategies of player j in DG, and the payoff function of j is $u^j: \Sigma \to R$, where $\Sigma = \times_{j \in F} \Sigma^j$. We say that a strategy profile σ is an equilibrium in DG if and only if it is an equilibrium in the associated game in strategic form.

Special Types

A distributed game is called a *parallel game* if each permutation is equally likely, that is, $\lambda(r) = (1/n!)$ for every permutation r, where n denotes the number of locations. This definition reflects the asynchronous nature of distributed systems: In a parallel game the interactions are supposed to be completely parallel, but due to the asynchronous nature of distributed systems there is a stochastic noise, implying sequential interactions. The players are ignorant about the precise nature of the noise and therefore, because all locations are symmetric, all they can do is use the principle of indifference (Keynes, 1963). That is, they assign equal probabilities to all permutations of locations. Using distributed computing terminology one can associate each location with a processor, and each agent with a process of a particular user. Each process has access to the clock of the processor where it runs, while the clocks of the different processors are not synchronized, as in typical asynchronous distributed systems. A distributed game is called *location-symmetric*, if all stage games are identical. That is, $G_i = G_l$ for all locations $1 \le i, l \le n$. In a location-symmetric distributed game it is useful to define the concept of location-free strategies. Let DG be a location-symmetric distributed game. Denote by S_c ("c" for "common") the set of action profiles in any location. A strategy σ^j is *location-free* if $\sigma_i^j(i, y)$, $y \in Y^a$, does not depend on i. That is, σ^j is location-free if there exist functions $f: Y^a \to S_c$ and $g: Y^a \times S_c \to M^j$ such that for every location i, $f_i^j(i, y) = f(y)$ and $g_i^j((i, y), x_c) = g(y, x_c)$. In our software agents terminology, a player who uses a location-free strategy sends n copies of the same software program rather than sending n distinct programs. In a location-symmetric distributed game, we say that σ is a *location-free equilibrium* if it is an equilibrium and σ^j is a location-free strategy for every player j.

Game Theoretic Features

It may be useful to summarize some special game theoretic features of distributed games. Such games can be described as extensive games with simultaneous moves (see, e.g., Osborne and Rubinstein, 1994) with incomplete information. They possess the additional feature that the information monitoring is partially controlled by the players via the messages: Classical repeated games in which the players observe at each stage the whole history of previous moves are referred to as games with perfect monitoring. Repeated games with imperfect monitoring are of two types: At the first type, the structure of the information released to the players after each stage does not depend on their actions. This is the case, e.g., in repeated games with observable payoffs and unobserved actions in which after each stage the players observe the history of all previous payoffs. Another example is games with bounded recall (Lehrer, 1988), in which after each stage the player observes the last k moves. In the other type of imperfect monitoring, the structure of monitoring depends on the actions. Such is the case in distributed games in which the structure of information release regarding a previously active location depends on whether one of the agents at this location sent a message.[10]

A distributed game (with at least two locations) has imperfect recall[11]: An agent of player j may not remember the actions chosen at previous locations, though the agent of player j at such previous location has observed some of these actions. Like the concept of monitoring, there are two types of recall structure. In one of them (as in games with bounded recall) the level of recall is action-independent. In the other type (e.g., in a distributed game) the level of recall depends on the actions. In particular, in distributed games the use of agents enables the players to jointly com-

[10]Note that every player has a strategy which guarantees that at every history which is consistent with this strategy, each of his agents knows the sequence of previous active locations. In particular, each agent knows the true stage at every stage. In such a strategy every agent of this player at every location broadcasts some message for every information signal it receives. However, since all players have symmetric information structure, using such a strategy implies that at every location, the sequence of past active locations is commonly known by the agents of *all* players. Note moreover that if the message space of player j contains a sufficient number of distinct messages (that is, if the language of player j's agents is sufficiently rich), then player j has a strategy which guarantees that it deduces, along every consistent history, the locations that have already been played and the actions and messages chosen at these locations.

[11]The concept of imperfect recall and related topics were already discussed in the early game theory literature, e.g., in Kuhn (1953), Dalkey (1953), Isbell (1957), and Aumann (1964). The original definition of perfect and imperfect recall (see, e.g., Mertens *et al.* 1994) applies to extensive games in which every decision node belongs to a single player. This definition is naturally generalized to extensive games with simultaneous moves and,in particular, to distributed games.

mit themselves (by not sending messages) not to remember that they have already been in a particular location.

Recently, following Piccione and Rubinstein (1997), equilibrium concepts in games with imperfect recall have been extensively discussed in the literature. See, e.g., the special issue on imperfect recall (Kalai, 1997). In particular, there is no agreement about the "right" concept of equilibrium in such games. Moreover, even the concept of "optimal" strategy in one person game with imperfect recall is not agreed upon. The debate, however, concerns abstract games. Our definition of equilibrium seems to us natural in distributed games in which the players send their agents (programs) to the locations before the game begins and they cannot replace them after the game begins.

Our definition of distributed games can be modified in several natural ways in order to deal with other types of distributed interactions. We discuss such modifications in Section 6.

3. COOPERATION IN THE PARALLEL PRISONER'S DILEMMA GAME

In this section we analyze a location-symmetric parallel game with two players and n locations, $L = \{1, \ldots, n\}$, in which at each location the agents of the players play the Prisoner's Dilemma game described below[12]:

$$\begin{array}{cc} & \begin{array}{cc} D & C \end{array} \\ \begin{array}{c} D \\ C \end{array} & \begin{pmatrix} (a, a) & (b, 0) \\ (0, b) & (c, c) \end{pmatrix}, \end{array}$$

where $b > c > a > 0$.

As we deal with a parallel game, the probability distribution λ assigns probability $1/n!$ to each order of locations. The message sets are M^j, $j = 1, 2$. We denote the empty message[13] by φ. We assume that each message space contains at least one message (hence, each M^j contains at least two elements, φ and a real message).

Consider the following inequality:

$$b \leq c + \frac{n-1}{2}(c - a). \tag{EC}$$

We show that if (EC) is satisfied, then there exists a location-free equilibrium $\sigma = (\sigma^1, \sigma^2)$ which induces the outcome (C, C) at each location.

[12] Our discussion and results hold for other forms of the Prisoner's Dilemma. In fact, our results can be extended to a much larger class of settings, as we prove in the following section.

[13] Recall that the empty message is not a message, it is just a way to describe the event that an agent does not send a message.

Indeed, let a^1, a^2 be nonempty messages in M^1, M^2, respectively. For $j = 1, 2$, define $\sigma^j = (\sigma_i^j)_{i=1}^n = ((f_i^j, g_i^j))_{i=1}^n$ as follows: f_i^j assigns the action C to the information signal (i, NM) of no previous messages and it assigns the action D to any other information signal in Z_i^a. In addition, independently of the information signal $z \in Z_i^a$, $g_i^j(z, (C, C)) = \varphi$, and $g_i^j(z, (C, D)) = g_i^j(z, (D, C)) = g_i^j(z, (D, D)) = a^j$. Obviously if all agents obey the strategy profile σ, then the outcome (C, C) is played in every location. We proceed to show that player 1 does not have a profitable deviation. The analogous proof for player 2 is omitted.

Let $\tau^1 = (\tau_i^1)_{i \in L} = (f_i^\tau, g_i^\tau)_{i \in L}$ be any strategy of player 1. Obviously $u^1(\sigma^1, \sigma^2) = nc$. We proceed to show that

$$u^1(\tau, \sigma^2) \leq nc.$$

Let DEV_τ be the set of locations i at which the agent of player 1 chooses D when it does not receive any message. That is, DEV_τ is the set of all i for which $f_i^\tau(i, NM) = D$. If DEV_τ is empty, then player 1 cannot get more than nc because at each of the n locations either he chooses C (and get at most c) or he chooses D after it receives some real messages. In the latter case player 2 who sticks to σ^2 also chooses D, and thus player 1 gets a. Since $a < c$, he cannot get more than nc when he uses τ and player 2 uses σ^2. Assume that DEV_τ is not empty and let k, $1 \leq k \leq n$, be the number of locations in DEV_τ. Let $\tilde{t}$ be the random variable describing the first occurrence of a location from DEV_τ. That is, $\tilde{t} = t$, $1 \leq t \leq n$, if $r(t) \in DEV_\tau$ and $r(s) \notin DEV_\tau$ for $s < t$. Given that $\tilde{t} = t$, player 1 receives at most $t - 1$ times the value c, one time the value b, and $n - t$ times the value a. Therefore the expected payoff of player 1 given that $\tilde{t} = t$, denoted by v_t^τ, satisfies:

$$v_t^\tau \leq v_t = (t-1)c + b + (n-t)a. \tag{3.1}$$

If $v_t^\tau < v_t$, then either at some stage $s < t$ player 1 chooses to send a message, causing the other player to switch to D, or at some stage $s > t$, player 1 chooses c after receiving a message. Without loss of generality we can consider only deviations to strategies τ for which $v_t^\tau = v_t$. Therefore we can assume that according to τ , player 1's deviations involve the action selecting strategies and not the message selecting strategies. Moreover, it can be assumed that these deviations have the following form: the agents of player 1 use D at every location $i \in DEV_\tau$, independently of the information signal. The other agents of player 1 use their part of the strategy σ^1. Hence, the expected payoff of player 1 upon a deviation to τ is

$$v^\tau = \sum_{t=1}^n v_t p_t^\tau,$$

where $p_t^\tau = \mu_{(\tau,\sigma^2)}(\tilde{t} = t)$. From symmetry, player 1 receives the same expected payoff at two such strategies τ and τ' for which the sets DEV_τ and $DEV_{\tau'}$ contain k locations each. Therefore, we can denote by v^k the expected value of player 1 upon a deviation at k locations. That is,

$$v^k = \sum_{t=1}^{n} v_t p_t^k, \tag{3.2}$$

where $p_t^k = \mu_{(\tau,\sigma^2)}(\tilde{t} = t)$, for some strategy τ for which DEV_τ contains k locations.

We now show that the expected value of player 1 resulting from a deviation in k locations is nonincreasing in k. That is, $v^k \geq v^{k+1}$ for $1 \leq k \leq n-1$. Indeed, denote by F_t^k the probability of first defection at stage t or before. That is,

$$F_t^k = \sum_{s=1}^{t} p_s^k.$$

It is easily verified that F^{k+1} stochastically dominates F^k (that is, $F_t^k \leq F_t^{k+1}$ for every $1 \leq t \leq n$). Moreover, because $b > c > a > 0$, $(v_t)_{t=1}^n$ is an increasing sequence. Therefore (3.2) implies the desired monotonicity. Hence, it suffices to show that a deviation at precisely one location is not profitable, that is, $v^1 \leq nc$. Since $p_t^1 = (1/n)$ for all t, we get from (3.1) and (3.2) that

$$v^1 = \frac{1}{n}\sum_{t=1}^{n} v_t = \frac{1}{n}\sum_{t=1}^{n}\big(t(c-a) + na - c + b\big).$$

It follows that the equilibrium condition is

$$\frac{n+1}{2}(c-a) + na - c + b \leq nc.$$

That is,

$$b \leq c + \frac{n-1}{2}(c-a).$$

■

Thus the outcome of cooperation at all locations can be obtained in equilibrium if the number of locations is sufficiently large. The precise meaning of “sufficiently large” is given by (EC).

4. COOPERATION IN THE PRISONER'S DILEMMA GAME: RELATED LITERATURE

It is instructive to compare our result with other cooperation results for the Prisoner's Dilemma game, obtained by assuming finite sequentiality and bounded rationality.[14] Radner (1986) showed that when we are ready to settle for ε equilibrium, then when the number of repetitions increases the corresponding sets of ε equilibria allow longer and longer periods of cooperation. Kreps *et al.* (1982) showed that if we assume that with an arbitrary small but positive exogenous probability, one of the players is playing tit-for-tat rather than maximizing, then with sufficiently long repetition, all sequential equilibria outcomes are close to the cooperative outcome (see, however, Fudenberg and Maskin, 1986). Aumann and Sorin (1989) proved that when every player ascribes a small positive exogenous probability to his opponent being an automaton with bounded recall, then every equilibrium in sufficiently long repetition of the game is close in the payoff space to the cooperative outcome. Neyman (1985) deals with finitely repeated Prisoner's Dilemma games in which the players are restricted to use automata with a fixed number of states. When the number of stages is large relative to the number of states,[15] the automata cannot effectively count the number of previous stages, and as in our case, this ignorance regarding the number of stages enables cooperation (in all stages) in equilibrium. Hence in the automaton model the players are not able to process the available information, while in our model, the players jointly and strategically decide not to obtain this information. We think that our model is more realistic in the distributed systems (e.g., Internet) setup in which it is assumed that players broadcast messages and act in a parallel asynchronous setting, while controlling software agents with a tremendous counting ability. Zemel (1989) used the finite automata model of Neyman with the additional feature of allowing the players to send and receive messages. This additional feature allows cooperation by saturating the computational resources of the players and thus preventing them from utilizing complex strategies.

The recent papers of Neyman (1998) and of Nishihara (1997) are related to our work. Neyman (1998) proved that the players in a repeated Prisoner's Dilemma can (almost) reach the cooperation outcome if they do not share common knowledge regarding the true number of stages. Roughly speaking, our model exhibits a "real life" situation where such lack of common knowledge is possible. Note, however, that in our model this lack of com-

[14] The folk theorems (see e.g., Aumann and Shapley, 1994 and Rubinstein, 1979) show that cooperation is possible in equilibria of the *infinite* repeated game.

[15] Or relative to the number of states to the power of $1/\varepsilon$, when we are ready to settle for ε equilibrium.

mon knowledge is not exogenous, but it is the result of the strategic decisions of the players to let their agents and opponents be ignorant about the number of (future) stages. Nishihara (1997) modified a one-stage N-person Prisoners' Dilemma game to a game in which the players move sequentially (i.e., there are N stages). He showed that if the players are randomly ordered according to the uniform distribution of orders, and at each stage each player receives a signal of defection (if such a defection occurred in previous stages), then the players can cooperate in equilibrium. Note that the signals in Nishihara's model are exogenously determined, while in our model they are determined by the players. In Nishihara's model there are no simultaneous moves, and therefore the concept of common knowledge is reduced to the concept of knowledge. In this sense, the lack of common knowledge of the number of future stages enables cooperation in his model. As in Neyman's model (and unlike our model) this lack of common knowledge is exogenous. Finally, the phenomenon of imperfect recall which is essential in our model does not appear in Neyman's model or in Nishihara's.

As for the structure of the strategy that leads to cooperation, one may wish to notice that it has some similarity with the grim strategy (see Osborne and Rubinstein (1994) for a description of this strategy in the context of the Prisoner's Dilemma). However, the exact strategy we employ relies heavily on the structure of distributed games, and makes use of its special features.

5. A GENERAL FOLK THEOREM

The results of Section 3 can be generalized to a folk theorem for location-symmetric parallel games.

THEOREM A. *Let G be a game in strategic form with the player set F and with the action sets S_c^j, $j \in F$. For every $n \geq 1$, let DG_n be the location-symmetric parallel game defined by $DG1 - DG6$ with $G_i = G$ for every $1 \leq i \leq n$. Let $b \in S_c = \times_{j\in F} S_c^j$, be an equilibrium in G. Let $x \in S_c$ satisfy*

$$u^j(x) > u^j(b) \quad \textit{for every} \quad j \in F.$$

Then, there exists a positive integer N such that for every $n \geq N$ there exists a location-free equilibrium σ in DG_n, at which the outcome x is reached at each location.

Proof. Let $a^j \in M^j$, $j \in F$ be an arbitrary selection of nonempty messages. For every $n \geq 2$ and for every j, define a location-free strategy $\sigma^j = \sigma(n)^j = (f^j, g^j)$ in DG_n, as follows: $f_i^j(i, NM) = x^j$, $f_i^j(z) = b^j$ for $z \in Z_i^a$ with $z \neq (i, NM)$, $g_i^j(z, x) = \varphi$ for every $z \in Z_i^a$, and $g_i^j(z, w) = a^j$

for every $z \in Z_i^a$ and for every $w \in S_c$ with $w \neq x$. It is obvious that if all players use their part in $\sigma(n)$, the outcome x is achieved at all locations. The proof that for sufficiently large n, $\sigma(n)$ is an equilibrium is similar to the analogous proof for the parallel Prisoner's Dilemma game, and therefore it is omitted. ■

6. REMARKS

Cryptography

The assumption that messages are not private information motivates some of cryptography theory (e.g., Diffie and Hellman, 1976). In the model of distributed games considered in this paper we do not distinguish between a message (e.g., a stream of bits) and its content. In more advanced models one may wish to deal with asymmetric information setup, where only the agents of the same player can decrypt their messages. The results proved in Sections 3 and 5 hold for such models too. Notice that if the language of messages is rich enough, agents can tell one another about their exact state. Hence, given cryptographic techniques, if agents of player i will send (encrypted) messages then the best they can do in this case is to send their full information. Our results show however that it may become strategically irrational to send any message.

Extensions

It is possible to extend the definition of distributed games in many ways. For example, one can replace the assumption that messages are not private information by the assumption that the messages can be received with some given probability distribution, that may be player-specific.

A more essential change in the definition of distributed game is obtained if we introduce some cost for every player who wishes his agents to receive messages sent by other players' agents.

Refinements

It can be easily verified that the equilibrium given in the proof of Theorem A is a sequential equilibrium, because the "punishing" action profile b is an equilibrium in the one stage game.

More General Folk Theorems

The proof of the folk theorem, Theorem A, is based on the existence of "punishing actions", b^j, $j \in F$, which are in equilibrium in the stage game. Proofs of the folk theorem for repeated games with complete information work without the requirement that the punishing actions are in equilibrium. However, in a repeated game with complete information, the players always know who were the deviators and therefore they can punish, say the first (in some arbitrary order of the players) to deviate. Such punishing strategies may not be feasible in parallel location-symmetric games, unless the message sets are rich enough. In the latter case, agents who observe a defection can reveal the identity of the "first deviator" by broadcasting an appropriate message.

The proof of Theorem A is based on the assumption that the agents are completely ignorant about the precise nature of the stochastic noise, caused by asynchronous clocks, and therefore they use the uniform distribution over permutations. When the locations are not symmetric (e.g., each is represented by a different type of processor), the players may have a better estimate about the nature of randomness. Therefore, it is interesting to investigate the class of (sequences of) distributions for which Theorem A is valid.

Modifications

Parallel games capture three main features of computerized distributed systems: Asynchronous clocks and activities, software agents,[16] and messages. Combined together, these features, translated to a game theoretic language, make the folk theorem work. Each of these features, on its own, and any two of these features give rise to a particular type of game. For example, one may wish to analyze distributed games with synchronized clocks. In such games the probability distribution λ is concentrated at a particular permutation. Such a game captures the features of agents and broadcasting.

Parallel Auctions

Our model of parallel games can be applied to parallel auctions, in which information about one auction may leak to other auctions. The folk theorem suggests that a collusion outcome may be obtained in equilibrium in such setting.

REFERENCES

Aumann, R. J. (1964), "Mixed and Behavior Strategies in Infinite Extensive Games," in *Annals. of Mathematics Study 52* (M. Dresher, L. S. Shapley, and A. W. Tucker, Eds.), Princeton: Princeton University Press.

[16] Or communicating sequential processes (see Hoare, 1985).

Aumann, R. J. and Sorin, S. (1989), “Cooperation and Bounded Recall,” *Games Econ. Behavior* **1**, 5–39.

Aumann, R. J. and Shapley, L. S. (1994), “Long Term Competition—A Game Theoretic Approach,” in *Essays in Game Theory, In Honor of Michael Maschler* (N. Meggido, Ed.), pp. 1–15.

Boutilier, C., Shoham, Y., and Wellman, M. P. (1997), Special Issue of *Artificial Intelligence*, on *Economic Principles of Multi-Agent Systems*, Vol. 94, 1997.

CACM (1994), “Intelligent Agents,” *Special Issue of CACM (Communications of the ACM)*, **37**(7).

Dalkey, N. (1953), “Equivalence of Information Patterns and Essentially Determinate Games”, in *Annals. of Mathematics Study 28* (H. W. Kuhn and A. W. Tucker, Eds.), Princeton: Princeton University Press.

Diffie, W. and Hellman, M. E. (1976), “New Directions in Cryptography,” *IEEE Transactions on Information Theory*, **22**(5), 644–654.

Fagin, R., Halpern, J., Moses, Y., and Vardi, M. (1995), *Reasoning About Knowledge*, Cambridge, MA: MIT Press.

Fudenberg, D. and Maskin, E. (1986), “The Folk Theorem in Repeated games with Discounting and Incomplete Information,” *Econometrica* **54**, 533–554.

Hoare, C. A. R. (1985), *Communicating Sequential Processes.* Englewood Cliffs: Prentice Hall.

Isbell, J. R. (1957), “Finitary Games,” in *Annals. of Mathematics Study 39* (M. Dresher, A. W. Tucker, and P. Wolfe, Eds.), Princeton: Princeton University Press.

Kalai, E. (1997) (Ed.), “Special Issue: Imperfect Recall,” *Games Econ. Behavior* **20**, 1–130.

Keynes, J. M. (1963), *A Treatise on Probability,* London: Macmillan.

Kuhn, H. W. (1953), “Extensive Games and the Problem of Information,” in *Annals. of Mathematics Study 28* (H. W. Kuhn and A. W. Tucker, Eds.), Princeton: Princeton University Press.

Kreps, D., Milgrom, P., Roberts, J., and Wilson, R. (1982), “Rational Cooperation in Finitely-Repeated Prisoners’ Dilemma,” *J. Econ. Theory* **27**, 245–252.

Lehrer, E. (1988), “Repeated Games with Stationary Bounded Recall,” *J. Econom. Theory* **46**, 130–144.

Mertens, J. F., Sorin, S., and Zamir, S. (1994), “Repeated games,” *Core Discussion Papers 9420, 9421, and 9422*, Universite Catholique De Louvain.

Neyman, A. (1985), “Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoner’s Dilemma,” *Econ. Lett.* **19**, 227–229.

Neyman, A. (1998), “Cooperation in Repeated Games when the Number of Stages is not Commonly Known,” *Econometrica* (forthcoming).

Nishihara, K. (1997), “A Resolution of N-person Prisoners’ dilemma, ” *Econ. Theory* **10**, 531–540.

Osborne, M. and Rubinstein, A. (1994), *A Course in Game Theory*, Cambridge, MA: MIT Press.

Piccione, M. and Rubinstein, A. (1997), “On the Interpretation of Decision Problems with Imperfect Recall,” *Games Econ. Behavior* **20**, 3–24.

Radner, R. (1986), “Can Bounded Rationality Resolve the Prisoners’ Dilemma,” in *Essays in Honor of Gerard Debreu* (A. Mas-Colell and W. Hildenbrand, Eds.), Amsterdam: North-Holland.

Rubinstein, A. (1979), “Equilibrium in Supergames With the Overtaking Criterion,” *J. Econ. Theory* **21**, 1–9.

Tanenbaum, A. S. (1988), *Computer Networks.* 2nd Ed., Englewood Cliffs, NJ: Prentice Hall.

Varian, H. R. (1995), "Economic Mechanism Design for Computerized Agents," Working paper, Berkeley University.

Zemel, E. (1989), "Small talk and Cooperation: A Note on Bounded Rationality," *J. Econ. Theory* **49**, 1–9.